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13. ABSTRACT *(Maximum 200 words)*

This research effort models decision-making behavior in a dynamic, uncertain environment. We derive consistent estimating procedures that model observed data that results from the decisions. This is accomplished by mating a Bayesian econometric process with a dynamic programming model of contractor behavior. This is used to demonstrate the dramatic increases in precision that can be obtained with just a few observations on program cost. The result of this effort can be used to develop a procedure for using routine data to help manage large programs in much the same way that control charts are routinely used to management production processes today.

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FINAL TECHNICAL REPORT

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OBJECTIVE:
To investigate approaches to dynamically generating cost-minimizing production plans in the made-to-order production scenario that is typical of defense contracting.

APPROACH:
In order to make production decisions to minimize the present value of *expected* cost while maintaining delivery schedules, we must be able to realistically model and estimate the relationship between production and cost. We are particularly interested in estimating parameters that determine returns to the variable factors and learning. Returns to the variable factors in a production situation refers to the tendency of the cost per unit to vary with the production rate. Typically cost increases at an increasing rate with the production rate, at least beyond some threshold rate. Learning refers to the cost savings gained from experience with the production process, with costs typically decreasing at a decreasing rate with cumulative production. These factors clearly affect the relationship between production level and cost.

It is important to estimate these parameters early enough in the production program to adjust the production schedule in anticipation of the pronounced effects of these factors. The production of a made-to-order product is often a non-recurring production process. As a result, the lack of prior experience creates a difficult problem for those interested in planning the production of the new product. While cost estimating relations and experience on prior programs may provide some guidance, prior production planning in this situation is uncertain at best without useful estimates for parameters of the production environment. But, waiting until there is enough data available from current production to make use of classical statistical estimation techniques may give results, but they are likely to be too late to be of practical use. This apparent dilemma eliminates the possibility of decision support from classical tools.

A nonlinear deterministic model is extended through the use of a bayesian updating scheme. Initial inputs for the parameters of the model are used to generate a cost-minimizing production schedule. The deterministic model and its parameters are defined as follows:

Decision Variables:

x_t = resources needed in time period t

q_t = production rate in time period t

Q_t = cumulative production for learning for period t production

Parameters:

γ = returns to variable factors parameter

δ = learning parameter

ρ = discount rate (assume discounting at the start of the period)

V = total volume required by the contract

T = ending period of the contract

B = a scale parameter

Minimize $\quad \sum_{t=1}^{T} x_t (1/(1+\rho))^{t-1}$ $\qquad\qquad\qquad$ (1)

subject to:

$$x_t = B q_t^{\gamma} Q_t^{\gamma\delta} \qquad\qquad t=1,2,...T \qquad\qquad (2)$$

$$Q_1 = q_1/2 \qquad\qquad\qquad\qquad\qquad (3)$$

$$Q_t = \sum_{k=1}^{t-1} q_k + (q_t/2) \qquad t=2,...T \qquad\qquad (4)$$

$$\sum_{t=1}^{T} q_t = V \qquad\qquad\qquad\qquad\qquad (5)$$

$$x_t, q_t, Q_t \geq 0 \qquad\qquad t=1,2,...T \qquad\qquad (6)$$

The objective function (1) minimizes the total discounted cost of resources used throughout the life of the program. Constraint set (2) defines resources used in a Cobb-Douglas type production function where resources are a function of current production rate and previous production experience. Constraints (3) and (4) define cumulative production (giving credit for half of the production in the current period. Constraint (5) requires that V units be produced in the T time periods of the program. Finally, (6) ensures non-negativity of the decision variables.

Through substitution and the use of an approximation for total resources utilized in period t, the problem may be written:

$$\text{Min } C = \sum_{k=1}^{T} B \left[\frac{(Q_{k-1} + q_k + 0.1)^{1-\delta} - (Q_{k-1} + 0.1)^{1-\delta}}{1-\delta} \right]^{\gamma} \left(\frac{1}{1+\rho} \right)^{k-1} \qquad (7)$$

subject to:

$$\sum_{t=1}^{T} q_t = V \qquad\qquad\qquad\qquad\qquad (8)$$

$$Q_t \geq 0 \qquad\qquad t=1,2,...T \qquad\qquad (9)$$

$$Q_0 = 0 \qquad\qquad\qquad\qquad\qquad\qquad (10)$$

Given a discrete joint probability distribution for γ, δ, and ρ, denoted $p(\gamma, \delta, \rho)$, the problem of minimizing *expected* discounted cost is given by:

$$\text{Min } C = \sum_{i=1}^{N} \sum_{k=1}^{T} B \left[\frac{(Q_{k-1} + q_k + 0.1)^{1-\delta} - (Q_{k-1} + 0.1)^{1-\delta}}{1-\delta} \right]^{\gamma} \left(\frac{1}{1+\rho} \right)^{k-1} p(\gamma_i, \delta_i, \rho_i) \qquad (11)$$

subject to:

$$\sum_{t=1}^{T} q_t = V \qquad (12)$$

$$Q_t \geq 0 \qquad t = 1, 2, \ldots T \qquad (13)$$

$$Q_0 = 0 \qquad (14)$$

Here, N is the number of discrete points of the joint distribution. The model (11)-(14) is utilized to plan the program so as to meet production requirements and minimize expected cost. We use this model dynamically over time to optimize the program given a new updated posterior distribution of the parameters. This updating scheme is conducted as follows:

We cast returns to the variable inputs and learning parameters as γ and δ, respectively. They relate cost (C_k') in period k to current and cumulative production in our model according to the following equation (Note that here we use undiscounted cost since that is what is observed.):

$$C_k' = B \left[\frac{(Q_{k-1} + q_k + 0.1)^{1-\delta} - (Q_{k-1} + 0.1)^{1-\delta}}{1-\delta} \right]^{\gamma} \qquad (15)$$

In each period, an observation on production cost is taken to update the model parameters. The parameters of C_k' are estimated with the relationship:

$$ACWP_k = C_k' \times \varepsilon_k \qquad (16)$$

where, $ACWP_k$ is an observation of the *actual cost of work performed*. ε is assumed to be log-normally distributed with a mean of 1. The q-vector is predicted from the probability distribution of the parameters and optimization based on information prior to the current observation. It should be noted that a classical approach would assume that the model parameters are fixed but simply unknown. In contrast, the Bayesian approach assumes that the unknown parameters are random variables, each with its own distribution. An immediate consequence is that the output of our analysis is a probability distribution rather than a point estimate.

The Bayesian updating process is described by:

$$Posterior_k(B, \sigma, \delta, \gamma) = \text{Pr}\,ior_k(B, \sigma, \delta, \gamma) * \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(z_k)}{\sigma}\right]^2} \qquad (17)$$

$$z_k = \frac{ACWP_k}{C_k} \tag{18}$$

where σ is the standard deviation of the error term. The term z_k is a ratio rather than a difference of the observation on cost for period k and C_k, due to the fact that we have opted for a multiplicative lognormal distributed error term in the estimating relationship.

ACCOMPLISHMENTS (throughout award period):
The optimization modeling and Bayesian updating procedure previously discussed were developed during the award period. Additionally, computer programs were written to perform these computationally intensive tasks. Neither the optimization nor the Bayesian updating is computationally trivial. The optimization model (11)-(14) was constructed in Microsoft Excel combining Excel Solver with Visual Basic for Applications. A barrier method was utilized when the Solver occasionally returned infeasible solutions (due to the discrete nature of the algorithm).

The Bayesian updating scheme is computationally intensive for realistic sized problems. Until recently, calculation of the posterior joint density function was not a reasonable task due to the tremendous processing power required. To prevent the posterior function from growing unmanageably complex, it is stored as a precise tensor of probabilities. To accurately represent a joint density function in a tensor, portions of the function, or specified ranges for each unknown, are numerically integrated with the goal of finding the probability of each of the variables falling within the specified ranges. Sufficient resolution of the ranges gives a satisfactorily close and maintainable representation of the posterior joint density for the unknowns. Therefore, updating the posterior involves a tremendous exercise of numerical integration.

The state of modern processing technology allows us to perform the necessary computation on a PC, but the number of variables with which we can deal remains severely limited. Our custom software, developed in ANSI C, calculated the 4-dimensional tensor of magnitude 100 in 11 minutes and 54 seconds on a 2.0 GHz PC with 512 MB RAM. For efficiency, the necessary magnitude to retain the desired precision for each dimension was determined by observing the sensitivity of the cost function to changes in the tensor.

The joint distribution yielded by this process permits the calculation of discounted expected program cost given some production schedule. Since the new distribution includes updated information about the production environment, it allows a more accurate calculation of the discounted program cost than possible in the previous period.

CONCLUSIONS:
We have developed an approach that combines nonlinear optimization with Bayesian updating for the dynamic planning of made-to-order production programs often found in defense contracting. This approach is new in that previous approaches have used static deterministic optimization to plan the program. The Bayesian approach proves useful in transforming a brief stream of observable data into continually improving information that can help make quality production decisions. An additional advantage of the Bayesian approach is the ability to make appropriate use of the knowledge and experience of managers and subject matter experts. Managers' subjective knowledge can be introduced in a meaningful way by developing a reasonable prior probability distribution for the model parameters. This may considerably improve the quality of the estimates in the first few periods. For our computational study, for the sake of objectivity, we chose an uninformed prior that is uniform. We

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found that the joint posterior distribution for the model parameters began to take a meaningful shape (clearly identifiable peak) after only two monthly observations of cost.

The work conducted under this grant is seminal work in the area of dynamic Bayesian optimization for made-to-order production planning. Much work remains to be done. First, the current work still relies on the manual running of two separate programs (the optimization model and the Bayesian updating model). To be useful in the field, these programs need to be integrated and the computational efficiency needs to be improved. Second, more detailed microeconomic models (for example, a production line model) combined with the Bayesian updating will be even more useful to managers.

SIGNIFICANCE:
Our study evinces that meaningful and timely estimates can be obtained after only a few periods using a Bayesian approach to estimate unobservable environment parameters. Where classical techniques fail to be timely, a Bayesian approach proves useful in transforming a brief stream of observable data into continually improving information that can help make quality production decisions.

PATENT INFORMATION:
No patents have been filed from this work.

AWARD INFORMATION:
No awards were given.

PRESENTATON OF THIS WORK (for total award period):
This work was presented as "Bayesian Program Management" at the 2nd Joint WINFORMS Symposium 22 and 23 April in Washington, D.C.
Several papers are in preparation.